ISBN: 978-981-18-5852-9
WCSE 2022 Spring Event: 2022 9th International Conference on Industrial Engineering and Applications
doi: 10.18178/wcse.2022.04.077

Analysis of the Influence of Atmospheric Haze on Quantum Satellite Communication Multi-Channel Performance

Feng Tian[1], Min Nie[1], Guang Yang[1], KaiJie Han[1] and JingQi Li[1]

[1] Xi 'an University of Posts and Telecommunications, School of Communication and Information Engineering, Xi 'an, China

Abstract. In quantum satellite communication, when the optical quantum signal is transmitted between satellite and ground links, it will be affected by various natural environments. At the same time, haze has become one of the largest environmental problems in the world. Therefore, it is of great significance to study the communication performance of quantum satellites in haze atmosphere. In this paper, according to the scale distribution and vertical spatial concentration change of large aerosol haze particles, the extinction characteristic model of large aerosol haze particles and the attenuation model of atmospheric haze particle concentration and transmission height on quantum satellite ground link are established. Some important performance parameters in quantum satellite communication are analyzed for amplitude damping channel, depolarization channel and phase damping channel respectively, It includes the quantitative relationship between channel capacity, quantum fidelity, quantum bit error rate and entanglement and large aerosol haze particle concentration and transmission height. The simulation results show that with the increase of transmission distance and haze particle concentration, the channel capacity, quantum fidelity and entanglement of quantum satellite communication in different channels will decrease, and the bit error rate will increase. Therefore, the impact of large haze environment on quantum satellite communication should be fully considered in the actual process of quantum satellite communication, Adaptive adjustment of relevant parameters to ensure the normal operation of quantum satellite communication.

Keywords: Quantum satellite communication, The aerosol haze, The channel capacity, Quantum bit error rate, fidelity

1. Introduction

With the continuous breakthrough of quantum communication technology in various countries around the world, the construction of global wide-area quantum communication network is an important strategic goal in the field of quantum communication in China and even the world at present, In the process of constructing the global wide-area quantum communication network, quantum fiber communication is limited to the ultra-high attenuation of long-distance transmission, so the use of quantum satellite to construct the communication network relay has become an indispensable and important link. The world's first quantum satellite has been successfully developed by the Chinese Academy of Sciences and launched in Jiuquan at 1:40 am on August 16, 2016. And in a few years after the successful launch, China has made breakthroughs in the quantum communication experiment of ultra-long distance quantum information transmission, and achieved excellent experimental results. Yet because when quantum between satellite and ground station in the process of communication, transmit information carrier of photons through its star to link will inevitably suffer from various atmospheric environment in the free space, so the quantum experiment satellite can only in good weather night, academician of Chinese Academy of Sciences professor jian-wei pan pointed out that the next generation of quantum satellite communications will implement all-weather work, But there are still some technical bottlenecks. Therefore, studying the influence of atmospheric environment on quantum satellite communication is an important step to solve the all-weather communication of quantum satellite and provides a reference for how to adjust various indicators in the quantum satellite system.

2. Aerosol extinction model and link attenuation

Haze is a general term for fog and haze. Fog particles are generally larger in radius than haze particles, and the size of fog droplets is larger than haze particles. The particle diameter is mostly between 430μm, and the diameter of haze particles is mostly between 0.01μm and 10μm, and the average diameter is about 1μm to 2μm. Fog is tiny water droplets, haze is fine solid particles. And the small droplets of fog also need small solid particles as condensation nuclei. Haze is studied as a whole in this paper.

The distribution formula commonly used for haze particles and cloud particles is the particle scale spectrum exponential distribution proposed by Deirmendjian, also known as the generalized gamma distribution. It has the following form:

$$n(r) = \frac{dN(r)}{dr} = Ar^{\alpha} \exp(-br^{\beta}) \tag{1}$$

Where, N is the number of particles in unit volume; $n(r)$ is the number of haze particles in unit volume and unit radius interval; A, b, α, β is a normal number; r is particle radius.

Haze in essence belongs to the category of aerosols, is made up of many, many, many shapes, the uneven distribution of tiny particles, the particles are distributed in the atmosphere of People's Daily life, they can to incoming electromagnetic waves produce certain reflection, absorption, scattering and refraction effect, so that the intensity of the incident light is weakening. In order to facilitate the study, it is assumed that the haze particles are homogenous and uniform spherical particles, so it is applicable to the Mie scattering theory. According to Miesian scattering theory, the scattering parameter formula of haze particles can be obtained:

$$Q_t(m,x) = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\left\{\mathrm{Re}(a_n + b_n)\right\} \tag{2}$$

$$Q_s(m,x) = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)(|a_n| + |b_n|) \tag{3}$$

$$Q_a(m,x) = \frac{1}{x^2}\left|\sum_{n=1}^{\infty}(2n+1)(-1)^n(a_n - b_n)\right|^2 \tag{4}$$

Where, Q_t, Q_S, Q_a are extinction efficiency factor, scattering efficiency factor and absorption efficiency factor respectively. a_n and b_n are the Mie scattering coefficients, $x = 2\pi r/\lambda$ is known as dimensional parameters. Although The Mie scattering theory is a very effective theory to solve particle scattering, its solution is relatively complicated. The approximate formula given by Vander Hulst can be used to solve the extinction factor Q_t quickly, and the expression is:

$$\begin{aligned} Q_t &= -4\exp(-a\tan b)[\frac{\cos b}{a}\sin(a-b) + (\frac{\cos b}{a})^2\cos(a-2b)] \\ &+4(\frac{\cos b}{a})^2\cos 2b + 2 \end{aligned} \tag{5}$$

$$a = 2x(n_r - 1), b = \tan^{-1}(\frac{n_i}{n_r - 1}) \tag{6}$$

Where, n_r represents the real part of the complex refractive index, and n_i represents the imaginary part of the complex refractive index.

The ground extinction coefficient can also be obtained:

$$\beta_t(0) = \int_{r_2}^{r_1} \pi r^2 Q_t \frac{dN}{dr} dr \tag{7}$$

Where, β_t represents extinction coefficient and dN/dr particle spectral density.

In the actual process of quantum satellite communication, since it is satellite-ground link transmission, oblique transmission should be considered as shown in the figure1.

Where, l represents the horizontal transmission distance, h represents the vertical transmission height, and θ refers to the outgoing Angle of the optical quantum signal at the transmitting end. The minimum height Angle for establishing connection between quantum satellite and ground station is generally in 30°. Therefore, the value range is $30^{\circ} \sim 60^{\circ}$. L is the transmission distance, then:

$$L = h \cdot scs\theta \tag{8}$$

In haze weather, the vertical spatial distribution of haze particle concentration decreases exponentially, as shown in the equation:

$$N_a(h) = N_a(0)\exp(-h/H_a) \tag{9}$$

Therefore, in the atmospheric haze environment, the oblique extinction coefficient of the optical quantum signal can be expressed as:

$$\beta_t = \beta(0)\exp(-h / H_a) \tag{10}$$

Here H_a refers to aerosol elevation, and the specific value corresponds to the visibility in haze weather, as shown in the following table 1.

The quantity concentration of haze particles can be expressed as:

$$N_a = 6w / \pi G r_c^3 10^{-9} \tag{11}$$

Where, N_a represents the quantity concentration, w represents the mass of particles per unit volume, G represents the specific gravity of particles, and r_c represents the mode radius of particles.

When the optical quantum signal sent by the transmitter end of the quantum satellite communication system is transmitted between the satellite-earth link and passes through the haze environment, the photon energy will be attenuated due to the influence of haze particles, so that the initial photon energy after passing through the haze environment can be written as follows:

TABLE I .Aerosol elevation value table

Visibility /km	2	3	4	5	6	8	10	13	25
H_a	0.84	0.90	0.96	0.99	1.03	1.10	1.15	1.23	1.45

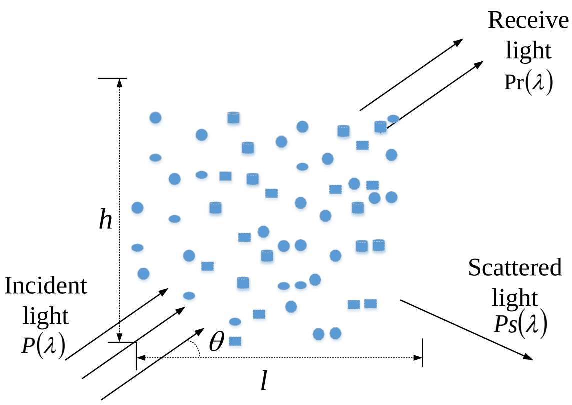

Fig. 1.Schematic diagram of oblique transmission

$$E = E_0 e^{-\beta_t L} \tag{12}$$

By taking logarithm of the above formula, link attenuation caused by haze particles can be expressed as:

$$A_t = 10 \cdot C_t \cdot \lg e \cdot L \tag{13}$$

Where A_t is the link attenuation caused by haze, C_t is the extinction section, which can be expressed as:

$$C_t = N_a \cdot \beta(h) \tag{14}$$

When the optical signal wavelength of quantum satellite communication is 860nm, the parameters are shown in Table 2.

we can see by the Fig.2 with the increase of concentration, optical signal through the fog haze layer by extinction effect increases leading to link attenuation increase gradually. In particle concentration under the condition of unchanged, the rise of the height, as the transmission link performance for when close to the ground to decay rapidly increased, with the rising of altitude, attenuation leveled off gradually, because of the fog in the troposphere, tropopause height is generally between 15 to 18 km so we can see for more than 10 km link attenuation gradually tend to be stable. When the particle concentration is $5\times10^{12}/m^3$ and the transmission height is from 0km to 1km, the attenuation of the link increases rapidly to 9dB, and when the transmission height is from 1km to 5km, the attenuati-on increases from 9dB to 17dB, and when the transmission height is from 10km to 15km, the attenuation increases from 17dB to 20dB. It can be seen that link attenuation is more obvious in the near surface process. Therefore, in practical quantum satellite communication, the influence of near-surface haze on signal attenuation should be given priority, and a better communication condition should be selected to reduce link attenuation and improve the quality of quantum communication by taking the concentration of ground haze as reference.

TABLE II. Parameter value table

A	α	b	β
0.00000498	2	15.1186	0.5

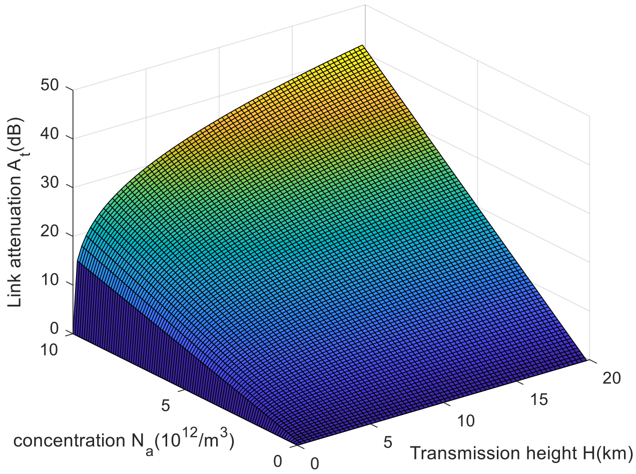

Fig. 2. The relationship between haze particle quantity concentration, transmission height and link attenuation

3. Influence of haze environment on different channel performance of quantum satellite communication

When the optical signal propagates in space, it is not only attenuated by the link, but also disturbed by the channel noise, which leads to the decrease of the coherence and entanglement of the optical quantum signal, which destroys the various properties of the quantum signal. There are several common channels in quantum communication, such as amplitude damping channel, depolarization channel, phase damping channel, etc. in this section, the channel parameters of each channel will be analyzed and simulated for different channels to explore the impact of haze particle concentration and transmission distance on various channel parameters.

A. Parameter analysis of amplitude damping channel

1.Amplitude damping channel capacity

Assume that the initial quantum state of haze particles is $|e_1\rangle$, When it jumps into a photon state $|e_0\rangle$ with the probability of losing a quantum of energy.During the transmission of the satellite-ground link,The quantum state of the system is $|\varphi_A\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$,Where, $|0\rangle$ represents the quantum ground state, and the excited state is expressed as $|1\rangle$, α_1, β_1 Is the probability of each state that meet the conditions is $|\alpha_1{}^2| + |\beta_1{}^2| = 1$,Thus, unitary evolution U_1 on the system formed by the combination of haze particles and quantum states can be expressed as:

$$U_1 : \begin{cases} |0\rangle_A|e_1\rangle \to |0\rangle_A|e_1\rangle \\ |1\rangle_A|e_1\rangle \to \sqrt{1-p_A}|1\rangle_A|e_1\rangle + \sqrt{p}|0\rangle_A|e_0\rangle \end{cases} \tag{14}$$

Where p_A is the probability of losing a photon, which can be expressed as:

$$p_A = 1 - \exp(\beta_t \cdot L) \tag{15}$$

It can be concluded that:

$$U\left(|\varphi_A\rangle|e_1\rangle\right) = \left(\alpha|0\rangle + \sqrt{1-p}\beta|1\rangle\right)|e_1\rangle + \sqrt{p}\beta|0\rangle|e_0\rangle \tag{16}$$

In the haze environment, with $\{|e_0\rangle, |e_1\rangle\}$ as the basis, the two Kraus operators of superoperator ϕ of the amplitude damping channel are:

$$\begin{cases} E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p_A} \end{pmatrix} \\ E_1 = \begin{pmatrix} 0 & \sqrt{p_A} \\ 0 & 0 \end{pmatrix} \end{cases} \tag{17}$$

Equivalent subsystem initialization density matrix is:

$$\rho_1^{'} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix} \tag{18}$$

After the optical signal passes through the amplitude damping channel, the state will change to:

$$\begin{aligned} \rho_2^{'} \to \varepsilon(\rho) \equiv \phi(\rho) = E_0\rho E_0 + E_1\rho E_1 \\ = \begin{pmatrix} 1-(1-p_A)(1-a) & b\sqrt{1-p_A} \\ c\sqrt{1-p_A} & c^*\sqrt{1-p_A} \end{pmatrix} \end{aligned} \tag{19}$$

Assume that the channel capacity of amplitude damping is C_A, and the source is $\{p_i, \rho_i\}$, p_i is the probability when the character is set to ρ_i. It can be known that $\sum p_i = 1$, assuming the input character $\rho_1 = |0\rangle\langle 0|, \rho_2 = |1\rangle\langle 1|$, then through the amplitude damping channel under haze weather, after the quantum state collision and entanglement of haze particles, the original quantum state evolution is as follows:

$$\begin{aligned} \rho_2^{'} &\to \varepsilon(\rho) = \varepsilon(\sum_i p_i\rho_i) \\ &= [p_1\rho_1 + (1-p_1\rho_2)] \\ &= \begin{pmatrix} p_1 + (1-p_1)p_A & 0 \\ 0 & (1-p_1)(1-p_A) \end{pmatrix} \end{aligned} \tag{20}$$

In quantum systems, von Neumann entropy is usually used to measure the quantum entanglement degree and the probability of information. The von Neumann entropy of the above process is:

$$\begin{aligned} S\left[\varepsilon(\sum_i p_i\rho_i)\right] = -[p_1 + (1-p_1)p_A]\,lb[p_1 + (1-p_1)p_A] \\ -(1-p_1)(1-p_A)^2\,lb(1-p_1) \end{aligned} \tag{21}$$

Usually, HSW theorem is used to calculate the capacity of some special noisy quantum channel, so that the amplitude damping channel capacity is:

$$\begin{aligned} C &= \max(S\left[\varepsilon(\sum_i p_i\rho_i)\right] - \sum_i p_i S[\varepsilon(\rho_i)]) \\ &= \max\begin{Bmatrix} -[p_1 + (1-p_1)p]\,lb[p_1 + (1-p_1)p_A] \\ -(1-p_1)(1-p_A)^2\,lb(1-p_1) - (1-p_1)H_2(p_A) \end{Bmatrix} \end{aligned} \tag{22}$$

Where $H_2(p)$ is Shannon's entropy.

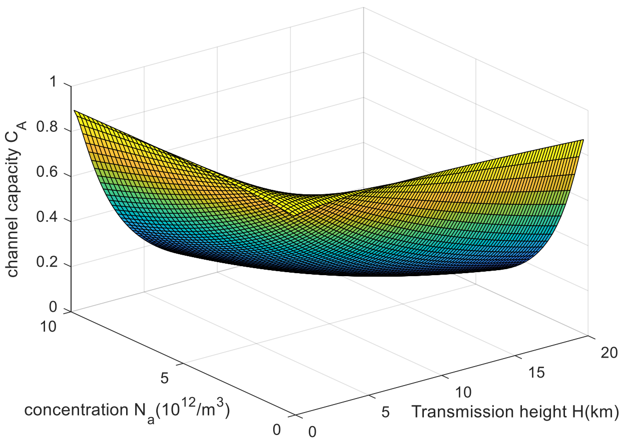

Fig. 3. The relationship between amplitude damping channel capacity and haze particle concentration and transmission height.

$$p_1 = \frac{\kappa(1-p_A)-p_A}{(1+\kappa)(1-p_A)} \tag{23}$$

$$\kappa = 2^{\frac{H(p_A)}{1-p_A}} \tag{24}$$

$$H(p) = -p\log p - (1-p)\log(1-p) \tag{25}$$

It can be concluded from the above formula that in the amplitude damping channel, the relationship between channel capacity, haze particle concentration and transmission height is shown in Figure 3. As can be seen from Figure 3, when the concentration of particles is constant, with the increase of signal transmission height, the optical signal is continuously subjected to the extinction effect of haze particles, and the link attenuation increases, leading to the gradual decrease of channel capacity. When the transmission distance remains unchanged, with the increase of particle concentration, quantum entanglement will occur between the quantum state of the quantum system and the haze environment, the coherence of the quantum state will be damaged, the probability of quantum bit error will increase, and the capacity of amplitude damping channel will also decrease gradually. Therefore, the influence of environmental factors on channel capacity should be fully considered in quantum satellite communication. As for the influence of haze on channel capacity, we can improve the reliability of quantum communication by increasing the bandwidth of quantum communication or the transmitting power of optical quantum signal.

2.Amplitude damping channel fidelity

In the process of quantum satellite communication, the coherence of quantum state will be damaged in haze environment, resulting in the decrease of the coherence of quantum bits. Channel fidelity can be used to measure the extent to which the channel maintains the state of the quantum system and reflect the ability of the quantum channel to transmit the quantum state effectively. When the quantum state of the haze particle is A_1 and the quantum state of the system is A_2, the fidelity F is defined as:

$$F\left(\rho_1^{'}, \rho_2^{'}\right) = Tr(\sqrt{\rho_2^{'1/2}\rho_1^{'}\rho_2^{'1/2}})^{1/2} \tag{26}$$

Where $\rho_1^{'}$ represents the final information density matrix and $\rho_2^{'}$ represents the quantum state density matrix of the system.

$$\rho_2^{'} = \begin{pmatrix} p_1 + (1-p_1)p_A & 0 \\ 0 & (1-p_1)(1-p_A) \end{pmatrix} \tag{27}$$

Assume that when conducting quantum satellite communication, the emitted quantum state is $|\varphi_A\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$, The initial density matrix of $|\varphi_A\rangle$ is shown in Equation () .The ground - to - satellite source is $\{p_i, \rho_i\}$. p_i is the probability when the character is ρ_i, and condition $\alpha = \beta = 1/\sqrt{2}$ is met, so the fidelity of amplitude damping channel can be expressed as:

$$\begin{aligned} F_A &= Tr(\sqrt{\sqrt{\begin{pmatrix} p_1+(1-p_1)p_A & 0 \\ 0 & (1-p_1)(1-p_A) \end{pmatrix}}\begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}\sqrt{\begin{pmatrix} p_1+(1-p_1)p_A & 0 \\ 0 & (1-p_1)(1-p_A) \end{pmatrix}}})^{1/2} \\ &= \sqrt{p_1(p_1+(1-p_1)p_A)} + (1-p_1)\sqrt{1-p_A} \end{aligned} \tag{28}$$

Combined with the above formula, when the optical signal transmits the signal at wavelength 0.86 μm, the relationship among haze particle quantity concentration, transmission height and fidelity can be obtained, as shown in Figure 4.It can be seen from the figure that when transmission height is constant, the probability of quantum bit error flipping increases with the increase of haze particle number concentration, resulting in the attenuation of entanglement fidelity. When the concentration of haze particles is constant, with the increase of transmission height, the coherence of quantum state is damaged more seriously, the quantum information is lost more, and the entanglement fidelity is smaller. The quantum entanglement degree is related to the source probability, so the quantum states with different entanglement degree can be obtained by adjusting the probability of the source appropriately. Therefore, in order to ensure better entanglement properties of quantum signals during transmission, appropriate entanglement quantum states should be selected according to environmental conditions.

3.Amplitude damping channel bit error rate

According to BB84 protocol, the channel bit error rate can be expressed as:

$$E_Q = \frac{N_{error}}{N_{sift}} \tag{29}$$

N_{error} is the received bit error rate, and N_{sift} is the total received bit rate. Among them:

$$N_{error} = \mu\omega_a(1-\zeta)\exp(-2\omega_a)+\zeta \tag{30}$$

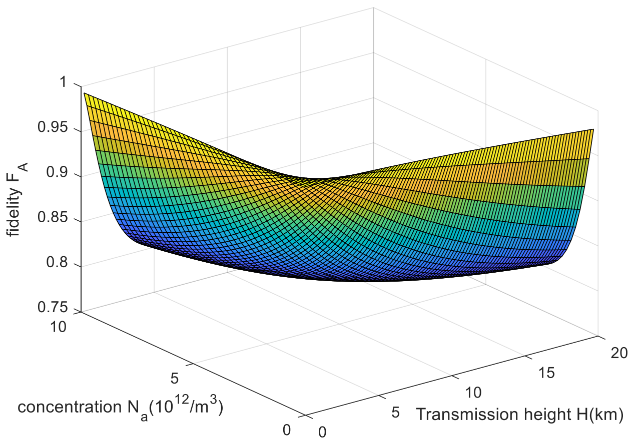

Fig. 4. The relationship between amplitude damping channel fidelity and haze particle quantity concentration and transmission height

μ represents the efficiency of quantum detection by photodetector, ω_b represents dark count caused by emitted photon and channel noise, ω_d represents dark current count of photodetector, and ζ represents depolarization effect factor.

$$N_{sift} = 4e^{\mu}F_s R_r(1-\exp(G)) \tag{31}$$

In the formula, F_s represents the screening factor, and R_r represents the photon pulse repetition rate.

$$G = \varsigma p R_a \mu F_c 10^{\sigma} \tag{32}$$

Where, ς represents the average photon number of light source pulse, p represents the single photon capture rate, R_a represents the transmission rate of the quantum system, and F_c represents the channel transmission factor, satisfying condition $\sigma = 0.1\beta_t\theta$.

The optical signal with wavelength 0.86 μm is used for simulation. The parameter values are based on Table 3, and the simulation results are shown in Figure 5.When the transmission distance and particle number concentration increase, the bit error rate increases. When the transmission distance and particle number concentration reach the maximum, the quantum bit error rate tends to 0.0059. When the transmission distance is constant, the bit error rate increases with the increase of particle concentration. This is because in the transmission process, the particles in the haze environment cause interference to the optical quantum state, which leads to the increase of bit error rate, the increase of the total bit error rate and the interference of key distribution, thus causing the deterioration of communication quality. When the concentration of particles is constant, the interference of the signal becomes larger with the increase of transmission distance, resulting in the bit error rate. Therefore, during key distribution, you must adjust parameters according to the communication environment to ensure communication quality.

TABLE III. Parameter value table

μ	ω_b	ω_d	F_s	R_r	ς	p
0. 65	10^{-3}	10^{-6}	0. 5	0. 5	1	0. 5

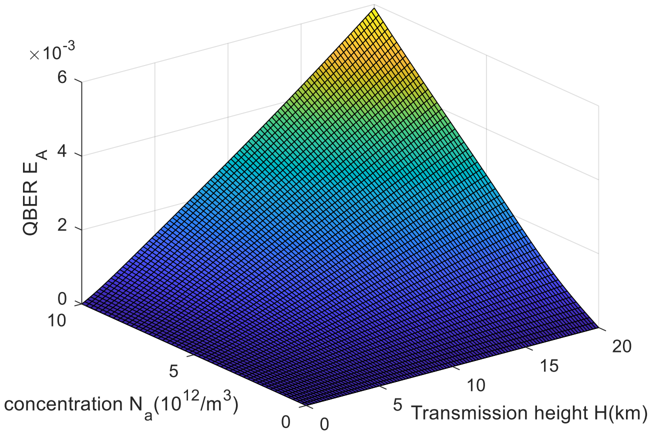

Fig. 5. Relationship among quantum bit error rate, haze particle concentration and transmission height

B. Parameter analysis in depolarized channel

1. Capacity of depolarized channel

In quantum satellite communications, the depolarization channel is one of the more common type of channel, its principle is in the process of quantum satellite communication, light quantum signal sent by a transmitter, when through the haze environment, due to the fog particles affect the qubits can produce out the possibility of a probability of p_B polarization phenomenon, to ignore the possibility that to maintain the original qubit.Among them, there are three error types with a probability of p_B, including bit flip, phase flip and mixed type, which can be expressed as:
$|e_1\rangle_B \to \sigma_1|e_1\rangle_B, |e_1\rangle_B \to \sigma_2|e_1\rangle_B, |e_1\rangle_B \to \sigma_3|e_1\rangle_B$.These three error types will cause errors in the transmission and processing of quantum information. When haze particles are entangled with qubits of light quantum signals, the depolarization effect generated by them can be expressed by unitary evolution as follows:

$$U_2 : |e\rangle \otimes |0\rangle_B \to \sqrt{1-p_B}|e\rangle \otimes |0\rangle_B + \sqrt{\frac{p_B}{3}}[\sigma_1|e\rangle \otimes |1\rangle_B + \sigma_2|e\rangle \otimes |2\rangle_B + \sigma_3|e\rangle \otimes |3\rangle_B] \tag{33}$$

Where, $|e\rangle$ is the quantum signal of the quantum system, and $|0\rangle, |1\rangle, |2\rangle, |3\rangle$ are both represented as the quantum state of haze particles in the haze environmental system. p_B represents the occurrence probability of depolarization effect of quantum signal, which can be expressed as:

$$p_B = 3/4(1-10^{-0.1\beta_t}) \tag{34}$$

After the optical signal depolarizes with the haze particles through the haze environment, the state can be expressed as:

$$\varepsilon(\rho) = (1-3p_B/4) + p/4(\sigma_1\rho\sigma_1 + \sigma_2\rho\sigma_2 + \sigma_3\rho\sigma_3) \tag{35}$$

For ρ, the relationship is as follows:

$$E/2 = (\rho + \sigma_1\rho\sigma_1 + +\sigma_2\rho\sigma_2 + \sigma_3\rho\sigma_3) \tag{36}$$

Thus, the formula can be obtained:

$$\varepsilon(\rho) = (1-3p/4) + p/4(\sigma_1\rho\sigma_1 + \sigma_2\rho\sigma_2 + \sigma_3\rho\sigma_3) \tag{37}$$

If character $\rho_1 = |0\rangle\langle 0|, \rho_2 = |1\rangle\langle 1|$ is entered, after the depolarization channel under haze weather, the quantum state collision and entanglement of haze particles, the original quantum state evolves into:

$$\varepsilon(\sum_i p_i\rho_i) = \varepsilon(p_0\rho_0 + (1-p_1)\rho_1) = \begin{pmatrix} p_B/2 + (1-p)p_0 & 0 \\ 0 & p_B/2 + (1-p_B)(1-p_0) \end{pmatrix} \tag{38}$$

Thus, the corresponding Von Neumann entropy as:

$$s[\varepsilon(\sum_i p_i\rho_i)] = -[(p_B/2+(1-p_B)p_0)]\log(p_B/2+(1-p_B)p_0)$$
$$+(p_B/2+(1-p_B)(1-p_0))$$
$$\log(p_B/2+(1-p_B)(1-p_0)) \quad (39)$$

The relationship between the channel capacity brought into the available depolarization channel and the haze particle concentration and transmission height is shown in Figure 6.

2. Depolarization channel fidelity

When the quantum state is transmitted in the depolarization channel under the haze background, the depolarization phenomenon caused by haze particles will lead to the decrease of the channel fidelity. According to the fidelity calculation formula, the final information density matrix of depolarization is:

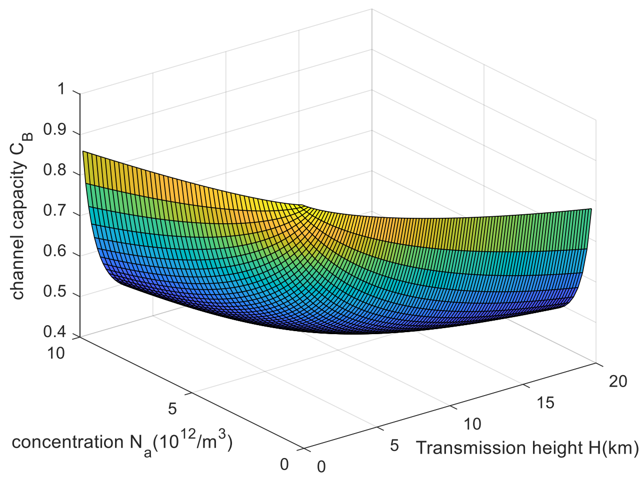

Fig. 6. Relationship among haze particle concentration, channel capacity and transmission height in depolarized channel

$$\rho_2^{'} = \begin{pmatrix} p/2+(1-p_B)p_0 & 0 \\ 0 & p/2+(1-p_B)(1-p_0) \end{pmatrix} \quad (40)$$

And the initial density matrix of information is:

$$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = \begin{pmatrix} |\alpha| & \alpha\beta^* \\ \alpha^*\beta & |\beta| \end{pmatrix} \quad (41)$$

Thus, the channel fidelity of the depolarized channel can be obtained by substituting:

$$F = Tr(\sqrt{\sqrt{\begin{matrix} p/2+(1-p_B)p_0 & 0 \\ 0 & p/2+(1-p_B)p_1 \end{matrix}}\begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha*\beta & |\beta|^2 \end{pmatrix}\sqrt{\begin{matrix} p/2+(1-p_B)p_0 & 0 \\ 0 & p/2+(1-p_B)p_1 \end{matrix}}}) \quad (42)$$

Finally, the fidelity of the depolarized channel can be obtained by simplifying the above formula:

$$F = \sqrt{\frac{p+2(1-p_B)p_1}{2}}\,p_1 + \sqrt{\frac{p+2(1-p_B)(1-p_1)}{2}}(1-p_1) \quad (43)$$

Through simulation, the relationship between channel fidelity of depolarization channel and haze particle concentration and transmission height is shown in Figure 7.

3. Bit error rate of depolarized channel

In the previous section, the paper analyses the amplitude damping channel quantum bit error rate is based on the BB84 protocol, this section based on decoy state communication protocol, the protocol is the BB84 protocol for further perfecting the security under the man-in-the-middle attack, the agreement is mainly the coherent light source approximate simulation single photon source emission, the transmitter sends a random phase of single photon pulse, The optical quantum signal sent can be expressed as:

$$\rho_A = \sum_{k=0}^{\infty} p_k |k\rangle\langle k| = \sum_{k=0}^{\infty} \frac{\mu^k}{k!}\exp(-u)|k\rangle\langle k| \quad (44)$$

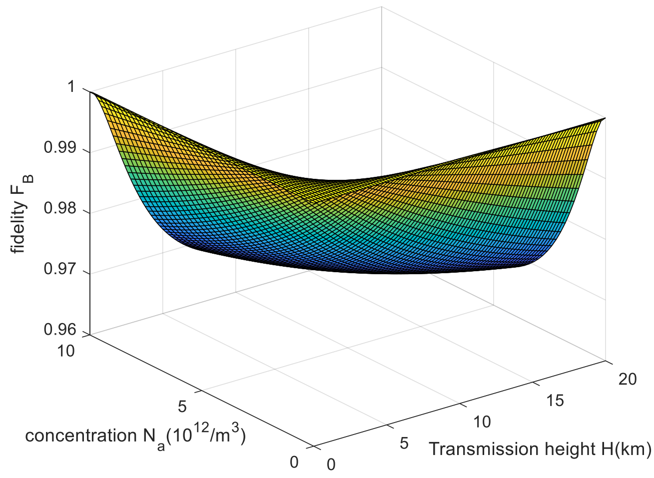

Fig. 7. Relationship among fidelity, haze particle concentration and transmission height in depolarized channel

The distribution of photon number in the transmitted optical quantum signal pulse obeys the Poisson distribution, that is: $p(k)=(\mu^k/k!)\exp(-\mu)$.In the decoy state protocol, if k = 0, known as the vacuum state, namely the light pulses sent in does not contain quantum state, namely the probability to $p(0)=\exp(-\mu)$, if k = 1, known as the single photon states, namely the light pulses sent in quantum state contains only a single photon, the probability of $p(1)=\mu\exp(-\mu)$, if k > = 2, called multiphoton states, namely the light pulses sent in quantum state contains multiple photons, the probability of $p(k\geq 2)=\sum_{k=2}^{\infty}p(k)=1-p(0)-p(1)$, From the above analysis, it can be seen that the selection of photon number intensity is very critical in the decoy state protocol. Here, the common value 0.1 is taken, and the distribution ratio of multi-photon states is about 0.5%, that is, the probability of K >2 is less than 0.5%. When it is necessary to calculate the ratio of single photon states in the transmitted optical quantum signal pulse, k<2 can be seen.

Therefore, in a section of optical pulse signal sent by the transmitter, the total quantum error rate of the channel is:

$$QBER=\sum_{k=0}^{2}e_kY_k\,p(k)=\sum_{k=0}^{2}e_kY_k\frac{\mu^k}{k!}\exp(-\mu) \tag{45}$$

Where, Y_k represents the counting rate of the vacuum state, single photon state and multi-photon state corresponding to k value. Its value is affected by the dark counting problem caused by noise in the atmospheric channel and the transmittance of optical quantum signal pulse 2. The relation can be expressed as follows:

$$Y_k=Y_0+\eta_k-Y_0\eta_k \tag{46}$$

e_k is the number error rate of photon states corresponding to each k photon states in the transmitted optical pulse signal, and its expression is:

$$e_k=[e_0Y_0+e_d(1-\eta_k)]/Y_k \tag{47}$$

Where e_0 refers to the error detection rate in dark count caused by channel noise, and e_d is the inherent error rate of quantum satellite communication system.

When the optical quantum signal pulse is transmitted in the environment of atmospheric haze, the value of η_k is affected by the transmission rate η_d of photon in the channel noise of atmospheric haze, and the relationship can be expressed as follows:

$$\eta_k=1-(1-\eta_d)^k \tag{48}$$

$$\eta_d=\exp(-r_d/\lambda_D)10^{-\beta_t\cdot h/100}\cdot\eta_b \tag{49}$$

Substitute into the formula to obtain:

$$QBER = e_0 Y_0 + \sum_{k=1}^{2} [e_0 Y_0 + e_d (1-\eta_d)^k] \cdot \frac{\mu^k}{k!} \exp(-\mu) \tag{50}$$

Table 4 Channel bit error rate Parameter Value

e_d	Y_0	e_0	η_b
3.3%	0.00000017	0.5	0.045

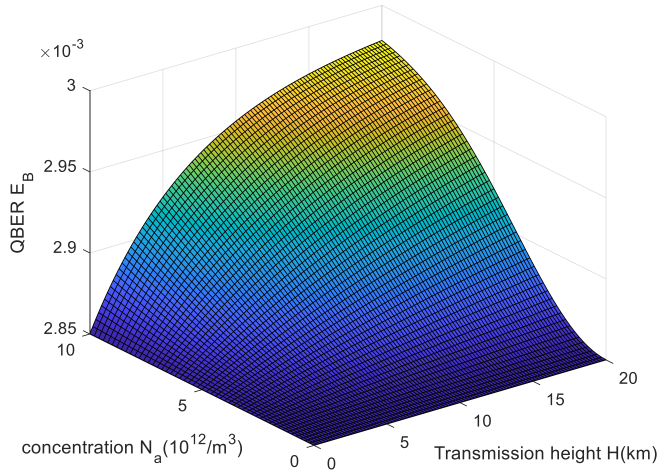

Fig. 8. Relationship among bit error rate, particle concentration and transmission height of decoy state protocol in depolarized channel

According to the above formula, the relationship between the channel error rate and haze particle concentration and transmission height of the depolarized channel can be obtained as shown in the figure 8 below, and parameter Settings are shown in Table 4.As can be seen from figure 6-8, the atmospheric haze environment will have a great impact on the channel parameters in the depolarization channel, including channel capacity, quantum fidelity and quantum bit error rate. With the increase of haze particle concentration and transmission height, the communication performance gradually decreases.

C. Parameter analysis in phase damping channel

1.Fidelity in phase damped channels

In the process of quantum satellite communication, the emitted optical quantum signal may experience the condition that its optical quantum does not have the energy transition, but will randomly change the phase of the optical quantum under the influence of environmental noise of atmospheric haze. In this case, the channel is called phase damping channel.

It is assumed that the quantum bits of the transmitted light quantum state in the initial state can be expressed as:

$$|\varphi_C\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle \tag{51}$$

When the light quantum and the haze particle in the haze environment scatter completely elastic, that is, the energy of the light qubit is lost in the process, and the probability p_C value and the probability of photon loss in the amplitude damping channel are similar to:

$$p_c = 1 - \exp(\beta_t \cdot L) \tag{52}$$

According to the literature, the two operation elements of the phase damping channel are respectively expressed as：

$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p_C} \end{pmatrix} \quad E_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p_C} \end{pmatrix} \tag{53}$$

When the phase of the light quantum state changes in the haze environment, the evolution process of the quantum state can be expressed as follows:

$$U_3\text{:}|e\rangle \otimes (a|0\rangle + b|1\rangle) \rightarrow (\sqrt{1-p_C}(a|0\rangle + b|1\rangle)|e\rangle I)$$
$$+(\sqrt{p_C}(a|0\rangle|e\rangle\sigma_x + b|1\rangle|e\rangle\sigma_y)) \tag{54}$$

Similarly, the information initial state density matrix is shown in equation (41).

Therefore, when the quantum state passes through the phase damping channel in the atmospheric haze environment, it can be expressed as:

$$\varepsilon(\rho) = \begin{pmatrix} |\alpha|^2 & \sqrt{1-p_C}\alpha\beta^* \\ \sqrt{1-p_C}\alpha^*\beta & |\beta|^2 \end{pmatrix} \tag{55}$$

Substitute in the fidelity formula to obtain:

$$F_C = Tr(\sqrt{\sqrt{\begin{matrix} |\alpha|^2 & \sqrt{1-p_C}\alpha\beta^* \\ \sqrt{1-p_C}\alpha^*\beta & |\beta|^2 \end{matrix}}\begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha*\beta & |\beta|^2 \end{pmatrix}\sqrt{\begin{matrix} |\alpha|^2 & \sqrt{1-p_C}\alpha\beta^* \\ \sqrt{1-p_C}\alpha^*\beta & |\beta|^2 \end{matrix}}}) \tag{56}$$

After simplification, the formula above can be obtained:

$$F_C = \sqrt{1+2p_1(1-p_1)(1-p_C)^{1/2}} \tag{57}$$

After simulation, the relationship between channel fidelity and haze particle concentration and transmission height of the depolarized channel can be obtained as figure 9.

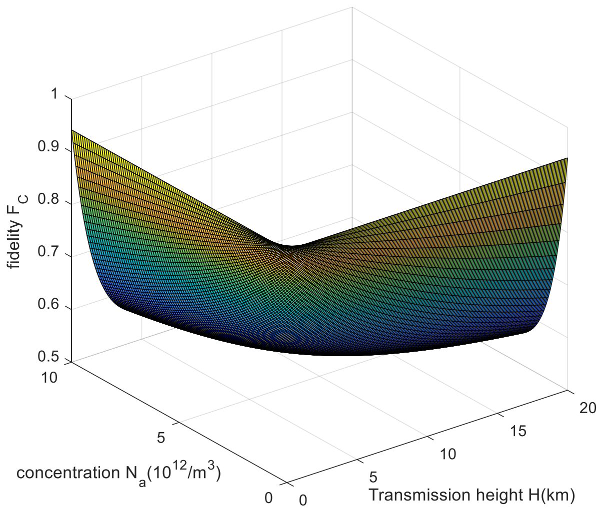

Fig. 9. Relationship among fidelity, particle concentration and transmission height in phase damped channel

2.Degree of entanglement in phase damped channels

Entanglement is an important parameter in quantum satellite communication. Only based on the properties of quantum entanglement can quantum teleportation and other related quantum communication technologies be successfully applied. In large aerosol haze environment, when the fog particles and photons scatter effect, can cause the quantum entanglement properties changed so that the degree of entanglement of the quantum of light signal is abate, in the process of quantum communication, the concurrence will have great influence on the communication performance, so this section explores the phase damping channel of aerosol haze environment light quantum state entanglement degrees.

According to the literature, the entanglement degree can be expressed as:

$$E(\rho_{AE}) = S(\rho_A) = S(\rho_E) = -\sum_i \lambda_i \log_2 \lambda_i \tag{58}$$

Where A refers to the quantum channel between the satellite and the ground, E refers to the atmospheric haze particle environment, $S(\rho_A)$ is the entropy of the quantum system, and $S(\rho_E)$ is the systematic entropy of haze particles in the atmospheric haze environment. ρ_{AE} is the density matrix of the quantum communication system after the evolution of haze environment, and λ_i is the eigenvalue of the density matrix.When the quantum state carrying the optical quantum signal is transmitted in the atmospheric haze environment, according to the evolution of the quantum state in the phase damping channel, the density matrix of the composite system of the environment and the quantum system can be expressed as:

$$\rho_{AE} = \sum_k \lambda_k |\psi_k\rangle\langle\psi_k| \tag{59}$$

Where, λ_k refers to the non-zero eigenvalues shared by the two systems in their respective reduced density matrices. $|\psi_k\rangle = \sum_i p_i \rho_i |e_s\rangle$ refers to the mixed state of the quantum state of the optical quantum signal and the state of the haze particles in the atmospheric haze environment; $|e_s\rangle$ is the final state formed by the interaction between the quantum state and the haze particles in the transmission process; if the information source is $\{p_i, \rho_i\}$, p_i is the probability that the character is ρ_i, $\sum p_i = 1$ can be obtained. Assuming that the input character is $\rho_1 = |0\rangle\langle 0|, \rho_2 = |1\rangle\langle 1|$and the scattering effect of haze particles is combined, the entanglement degree of the quantum channel can be defined as:

$$E = \frac{M \cdot H(p_0)}{\beta_t \exp(r_d / \lambda_D) \log(H)} \tag{60}$$

In the above formula, is the normalized constant take 20.1 as the average particle radius, assume that the particle radius is 200nm and the transmission distance is 20km, and simulate the relationship between haze particle concentration, source character probability and channel entanglement, as shown in the figure 10.It can be seen that the degree of entanglement has the maximum value when the character takes the equal probability, and the degree of entanglement will gradually decrease with the increase of transmission height.

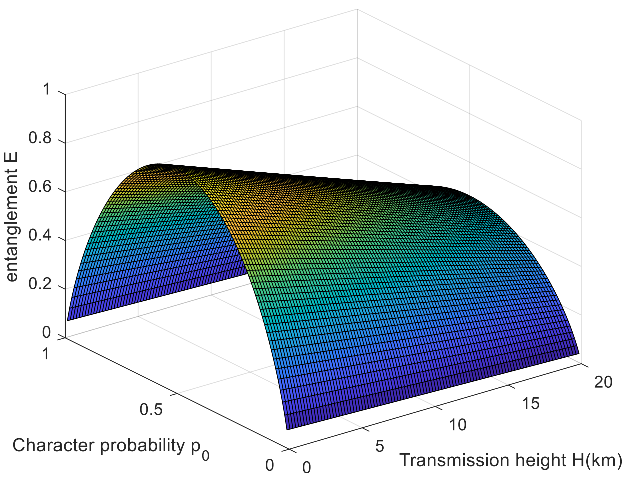

Fig. 10. Relationship among entanglement, particle concentration and transmission height in phase damped channel

4. Conclusion

In this paper, the performance parameters of quantum satellite communication in common channels are analyzed, including channel capacity in amplitude damping channel, quantum fidelity, quantum error rate based on BB84 protocol, mainly by establishing extinction characteristic model under the background of atmospheric haze. Channel capacity, quantum fidelity and quantum error rate based on decoy BB84 protocol in depolarization channel, quantum fidelity and entanglement in phase damping channel. Through the above simulation analysis, it is not difficult to see that quantum satellite communication performance will be seriously affected in the haze environment, so how to ensure the normal operation of quantum satellite communication in the haze environment is very important. In quantum satellite communication, feedback can be established between the receiver and transmitter, and the performance of quantum satellite communication can be improved by adjusting relevant parameters (brightness, wavelength, average photon number, etc.). In the follow-up, several reference methods will be provided, which will not be described in this paper due to the limited space.

5. References

[1] Liu X N,Hu C C,Yin M J,Jin L. "Simulation detection based on single photon quantum key distribution protocol". Journal of Physics: Conference Series 1812.1(2021): 012005-.

[2] C.Kurtsiefer,P.Zarda,M.Halder M,et al. "A step towards global key distribution.". Nature 419.6906(2002): 450.

[3] Yin Juan,Ren Ji-Gang,Lu He,et al. "Quantum teleportation and entanglement distribution over 100-kilometre free-space channels.". Nature 488.7410(2012): 185-8.

[4] N. Gisin and R.T. Thew. "Quantum communication technology". Electronics Letters 46.14(2010): 965-967.

[5] Zhao Hujia et al. "Climatological variations in aerosol optical depth and aerosol type identification in Liaoning of Northeast China based on MODIS data from 2002 to 2019". Science of the Total Environment 781(2021)

[6] Jin Zhi Xiang et al. "Maximum relative entropy of coherence for quantum channels". Science China Physics, Mechanics & Astronomy 64.8(2021)

[7] Giacomo De Palma and Dario Trevisan. "Quantum Optimal Transport with Quantum Channels". Annales Henri Poincaré 22.10(2021): 1-36.

[8] Gonzaga-Galeana J Alejandro and Zurita-Sánchez Jorge R. "Mie scattering of partially coherent light: controlling absorption with spatial coherence." Optics express 26.3(2018): 2928-2943.

[9] Attema Thomas and Bosman Joost W. and Neumann Niels M. P.. "Optimizing the decoy-state BB84 QKD protocol parameters". Quantum Information Processing 20.4(2021)

[10] Adel Zaghloul, Abdalhameed A. Shaalan, H. Kasban, and Amal Ashraf, "Evaluation of Free Space Optics Uplink Availability to LEO Satellite Using Climatic Data in Cairo," Journal of Communications vol. 16, no. 8, pp. 301-310, August 2021.

[11] NIE Min, REN Jia ming, YANG Guang,ZHANG Mei ling, . "Influences of the Ice-water Mixed Clouds on the Performance of Quantum Satellite Communication". ACTA PHOTONICA SINICA 45.9(2016): 927004-927004.

[12] Abdel-Haleem Abdel-Aty, Nordin Zakaria, Lee Yen Cheong, and Nasser Metwallay, "Quantum Network via Partial Entangled State," Journal of Communications, vol. 9, no. 5, pp. 379-384, 2014.